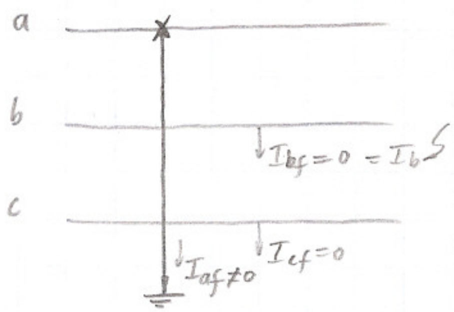
POWER SYSTEMS. FEB 14

Midterm ... March 9th. 1430 → 1600

Everything up to 3Ø faults (ass't 1-4).

Ex. Single line to ground fault ... (SLG)



assumed to denote fault current

It is assumed that the fault is on AØ only, which means that:

$$I_{bf} = I_b = I_{cf} = I_c = 0.$$

$$\underline{I_f} = \begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

In terms of symmetrical components:

$$\begin{bmatrix} I_a \neq 0 \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_a^{\circ} \\ I_a^{+} \\ I_a^{-} \end{bmatrix}$$

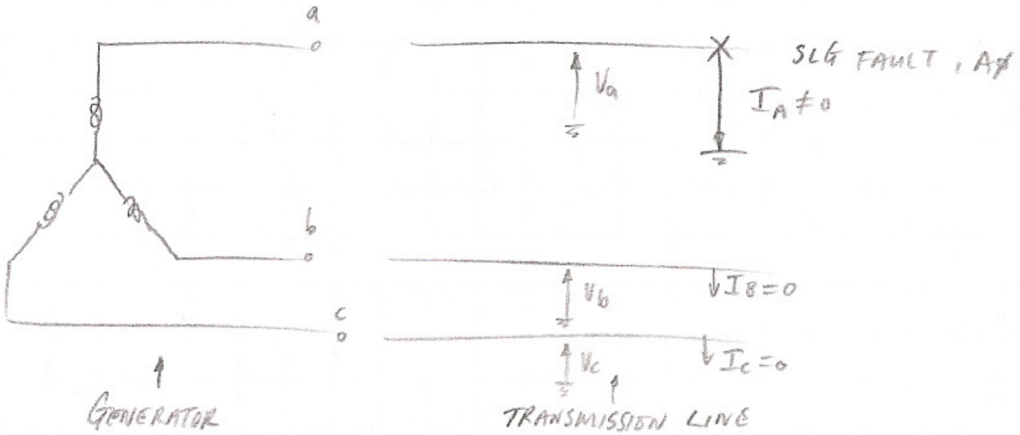
From above, $I_{af} = I_a^{\circ} + I_a^{+} + I_a^{-}$

We may also write

$$\begin{bmatrix} I_a^{\circ} \\ I_a^{+} \\ I_a^{-} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \longrightarrow \text{then:}$$

$$I_a^{\circ} = I_a^{+} = I_a^{-} = \frac{1}{3} I_a$$

E_{2}^{x}



Generators operate under balanced 3ϕ voltages which are only +ve sequence voltages. At the generator end we may write

$$E_a^+ = E_a$$
$$\left.\begin{array}{l} E_a^- = 0 \\ E_a^0 = 0 \end{array}\right\} \text{ due to balanced } 3\phi \text{ voltages}$$

Let us assume that the sequence impedances to the fault are Z^0, Z^+ and Z^-. Then we may say

$$V_a^0 = \underset{\substack{\uparrow \\ E_a^0 = 0}}{0} - Z^0 I_a^0$$

$$V_a^+ = E_a^+ - Z^+ I_a^+$$

$$V_a^- = \underset{\substack{\uparrow \\ E_a^- = 0}}{0} - Z^- I_a^-$$

Since phase a is grounded due to the fault,

$\underline{V_a = 0}$.

but $V_a = V_a^0 + V_a^+ + V_a^-$, and

$$0 = V_a = (-Z^0 I_a^0) + (E_a^+ - Z^+ I_a^-) + (-Z^- I_a^-)$$

NOTE: generator voltages have no zero or -ve seq. components

$$0 = E_a^+ - (Z^+ + Z^- + Z^0) I_a^0$$

$I_a^0 = I_a^+ = I_a^-$ see prev. page

for Aϕ-GND (SLG) Fault.

or: $$I_a^+ = I_a^- = I_a^0 = \frac{E_a^+}{Z^0 + Z^+ + Z^-}$$

In terms of phase currents...

$$\left(I_a^+ = \frac{1}{3} I_a \Rightarrow I_a = 3 I_a^+\right)$$

$$I_a = \frac{3 E_a^+}{Z^0 + Z^+ + Z^-}$$

$I_b = 0$
$I_c = 0$

For phase voltages, we may write

$$V_a = 0$$

$$V_b = \frac{E_b (1-\alpha)(Z^\circ + (1+\alpha)Z^-)}{Z^\circ + Z^+ + Z^-}$$

$$V_c = \frac{E_c (1-\alpha)((1+\alpha)Z^\circ + Z^-)}{Z^\circ + Z^+ + Z^-}$$

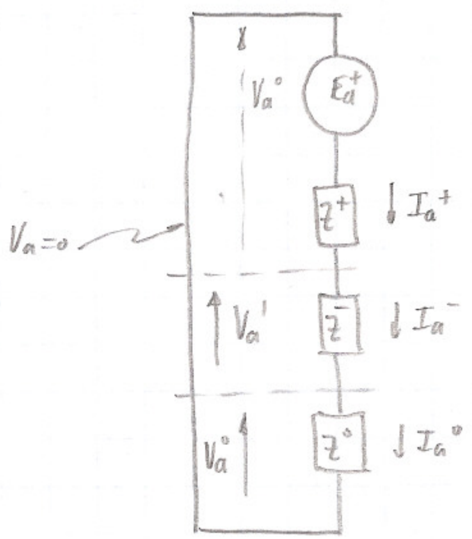
The last two expressions can be found from

$$V_b = V_a^\circ + \alpha^2 V_a^+ + \alpha V_a^-,$$
$$V_c = V_a^\circ + \alpha V_a^+ + \alpha^2 V_a^-$$

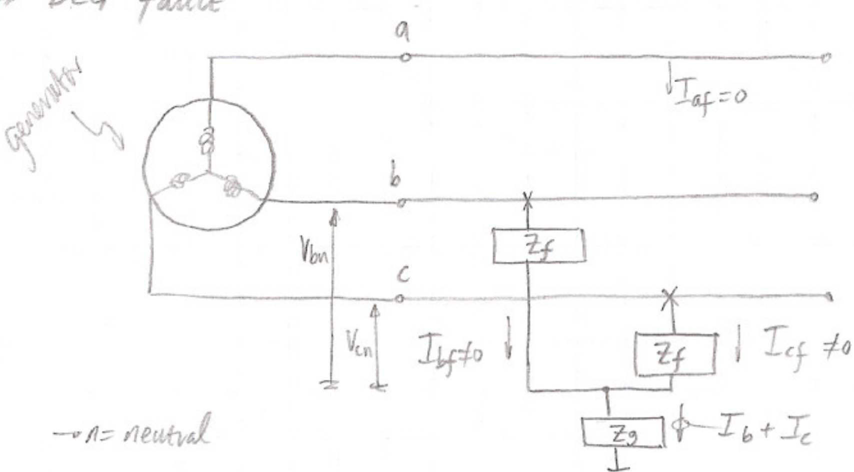
We may also write:

$$V_b = \frac{\alpha^2 E_a^+ - E_a^- (\alpha^2 Z^+ + \alpha Z^- + Z_0)}{Z^\circ + Z^+ + Z^-}$$

Sketching the equivalent cct for an Aϕ SLG fault:



⇒ DLG fault



In the above, we assume that phase B has a fault impedance Z_f and phase C has also an impedance Z_f and the common branch to ground has an impedance Z_g.

From the diagram,

$$V_{cn} = I_c Z_f + (I_c + I_b) Z_g$$

$$\text{or} \quad V_{cn} = I_b Z_g + (Z_f + Z_g) I_c$$

$$V_{bc} = V_{bn} - V_{cn} = I_b Z_f + \cancel{I_b Z_g} + \cancel{I_c Z_g} - \cancel{I_b Z_g} - I_c Z_f - \cancel{I_c Z_g}$$

$$\text{or} \quad V_{bc} = I_b Z_f - I_c Z_f = Z_f (I_b - I_c)$$

In terms of sequence currents and voltages,

$$V_{bn} = V_b = V_a^0 + \alpha^2 V_a^+ + \alpha V_a^-$$

$$V_{cn} = V_c = V_a^0 + \alpha V_a^+ + \alpha^2 V_a^-$$

$$I_b = I_a^0 + \alpha^2 I_a^+ + \alpha I_a^-$$

$$I_c = I_a^0 + \alpha I_a^+ + \alpha^2 I_a^-$$

$$\therefore V_a^0 + \alpha^2 V_a^+ + \alpha V^- - V_a^0 - \alpha V_a^- - \alpha^2 V_a^- = Z_f \{ I_a^0 + \alpha^2 I_a^+ + \alpha I_a^- - I_a^0 - \alpha I_a^+ - \alpha^2 I_a^- \}$$

$$\therefore (\alpha^2 - \alpha) V_a^+ + (\alpha - \alpha^2) V_a^- = Z_f \{ (\alpha^2 - \alpha) I_a^+ + (\alpha - \alpha^2) I_a^- \}$$

$$\text{or} \quad (\alpha^2 - \alpha) V_a^+ - (\alpha^2 - \alpha) V_a^- = Z_f \{ (\alpha^2 - \alpha) I_a^+ - (\alpha^2 - \alpha) I_a^- \}$$

$$\therefore \quad V_a^+ - V_a^- = Z_f (I_a^+ - I_a^-)$$

(A) $$\text{or} \quad V_a^+ - Z_f I_a^+ = V_a^- + Z_f I_a^- \qquad (A)$$

Adding V_b and V_c ..

$$V_b + V_c = I_b Z_f + I_b Z_g + I_c Z_g + I_b Z_g + I_c Z_f + I_c Z_g$$

$$\text{or} \quad V_b + V_c = (I_b + I_c)(Z_f + 2Z_g)$$

...

In terms of sequence quantities...

$$\alpha^2 V_a^+ + \alpha V_a^- + V_a^0 + \alpha V_a^+ + \alpha^2 V_a^- = \left[I_a^0 + \alpha^2 I_a^+ + \alpha I_a^- + I_a^0 + \alpha I_a^+ + \alpha^2 I_a^- \right](Z_f + 2Z_g)$$

Simplifying ...

D-L-G faults (cont'd)...

Simplifying...

$$2V_a^0 + (\alpha^2+\alpha)(V_a^+ + V_a^-) = [2I_a^0 + (\alpha^2+\alpha)(I_a^+ + I_a^-)][Z_f + 2Z_g]$$

$$\left.\begin{aligned}(\alpha^2+\alpha) &= -1.\\ (1\cdot e^{j240^\circ} + 1\cdot e^{j120^\circ}) &= -1\end{aligned}\right\}\text{ using this, then...}$$

$2V_a^0 - V_a^+ - V_a^- = [2I_a^0 - I_a^+ - I_a^-][Z_f + 2Z_g]$, but we know that

$I_a = I_a^0 + I_a^+ + I_a^- = 0$ (no fault on Aϕ)'s?

Using the (c)above, we get

$$2V_a^0 - V_a^+ - V_a^- = 3I_a^0(Z_f + 2Z_g) \quad (B)$$

From (A), we have

(C) $$V_a^- = V_a^+ - Z_f I_a^+ + Z_f I_a^- \quad (C)$$

But $I_a = 0$, then

$$0 = I_a^0 + I_a^+ + I_a^-$$

(D) or $$I_a^- = -(I_a^0 + I_a^+) \quad (D)$$

(D) $\to$ (C), we get:

$$V_a^- = V_a^+ - 2Z_f I_a^+ - Z_f I_a^0 \quad (E)$$

(E) $\to$ (B), and simplifying, we get

(F) $$V_a^0 - I_a^0(Z_f + 3Z_g) = V_a^+ - I_a^+ Z_f \quad (F)$$

We also have

(G) $$V_a^+ = E_a^+ - I_a^+ Z^+$$

...

$$(G) \qquad \left.\begin{aligned} V_a^- &= -I_a^- Z^- \\ V_a^0 &= -I_a^0 Z^0 \end{aligned}\right\} \qquad (G)$$

(G) → (F) and simplifying, we get

$$E_a^+ - I_a^+ (Z^+ + Z_f) = -I_a^0 (Z^0 + Z_f + 3Z_g)$$

or $$E_a^+ = I_a^+ (Z^+ + Z_f) + \underbrace{(I_a^+ + I_a^-)}_{-I_a^0} (Z^0 + Z_f + 3Z_g)$$

Simplifying, we get

$$(H) \qquad E_a^+ = I_a^+ (Z^+ + Z^0 + 2Z_f + 3Z_g) + I_a^- (Z^0 + Z_f + 3Z_g) \qquad (H)$$

From (A), (repeated here): (A): $V_a^+ - Z_f I_a^+ = V_a^- - Z_f I_a^-$

then

$$E_a^+ - I_a^+ (Z^+ + Z_f) = -I_a^- (Z^- + Z_f)$$

$$(I) \quad \therefore I_a^- = \left(\frac{I_a^+ (Z^+ + Z_f) - E_a^+}{Z^- + Z_f} \right) \qquad (I)$$

From (H):

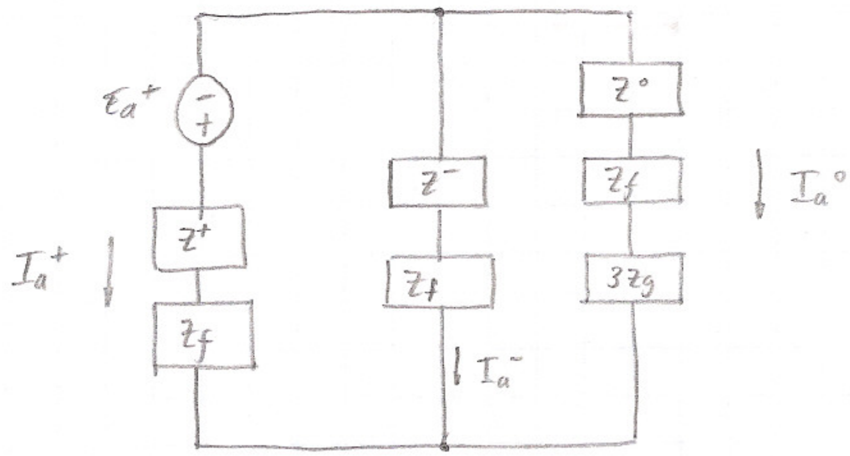
$$(J) \qquad I_a^- = \frac{E_a^+ - I_a^+ (Z^+ + Z^0 + 2Z_f + 3Z_g)}{(Z^0 + Z_f + 3Z_g)} \qquad (J)$$

(I) = (J) and simplifying...

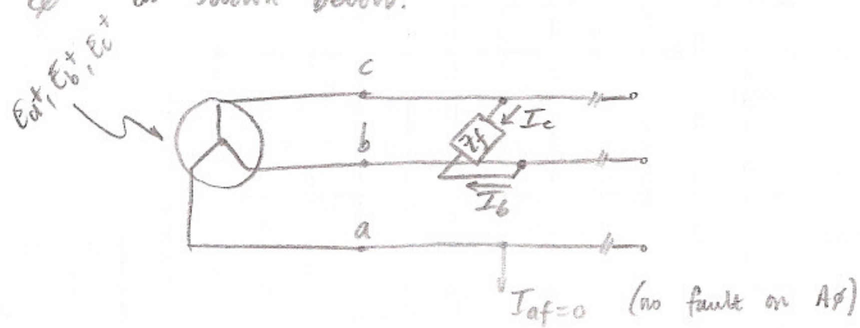
$$I_a^+ = \frac{\left[\dfrac{E_a^+}{(Z^+ + Z_f)(Z^0 + Z_f + 3Z_g) + (Z^- + Z_f)(Z^+ + Z^0 + 2Z_f + 3Z_g)} \right]}{(Z^- + Z^0 + 2Z_f + 3Z_g)}$$

Substitute into (J)...
We get an expression for I_a^- and I_a^0 from $I_a^0 = -(I_a^+ + I_a^-)$

From above, our equivalent circuit is



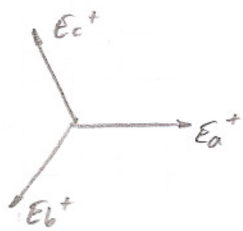
Ex: Sketch the equivalent circuit for a L-L fault between BØ and CØ as shown below:



From the diagram,

- $I_b + I_c = 0 \rightarrow I_b = -I_c$
- $I_a = I_a^0 + I_a^+, I_a^- = 0$ (fault current on AØ = 0)

Generator voltages:



(ABC Sequence)

$$I_a = I_a^0 + I_a^+ + I_a^-$$

$$I_b = I_a^0 + \alpha^2 I_a^+ + \alpha I_a^-$$

$$I_c = I_a^0 + \alpha I_a^+ + \alpha^2 I_a^-$$

Since $I_b + I_c = 0 \ldots$

$$(I_a^0 + \alpha^2 I_a^+ + \alpha I_a^-) + (I_a^0 + \alpha I_a^+ + \alpha^2 I_a^-) = 0$$

$$2I_a^0 + (\alpha^2+\alpha) I_a^+ + (\alpha+\alpha^2) I_a^- = 0$$

$$2I_a^0 - I_a^+ - I_a^- = 0 \quad \checkmark \qquad (I_a^0 = 0 \therefore I_a^+ = -I_a^-)$$

$(1\angle 240^\circ) + (1\angle 120^\circ) = 1\angle 180^\circ = -1$

$I_a^0 = 0$, from matrix form...

$$\begin{pmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{pmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a = 0 \\ I_b \\ I_c \end{bmatrix}$$

$I_a^0 = \frac{1}{3}((1)(0) + (1)(I_b) + (1)(I_c))$,

$I_b = -I_c \therefore I_a^0 = \frac{1}{3}(0) = \underline{\underline{0}}$

Actual Solution...

$I_a = 0$

$I_b = -I_c$

$V_b - V_c = Z_f \cdot I_b$

$I_a = 0 = I_a^0 + I_a^+ + I_a^-$ $\longrightarrow$ since $I_a^0 = \frac{1}{3}\{I_a + I_b + I_c\}$

and $\underbrace{I_a}_{0} + \underbrace{I_b + I_c}_{0} = 0$,

$\underline{\underline{I_a^0 = 0}}$

$I_a^+ = \frac{1}{3}\{\cancel{I_a} + \alpha I_b + \alpha^2 I_c\}$

$I_a^- = \frac{1}{3}\{\cancel{I_a} + \alpha^2 I_b + \alpha I_c\}$

$$\overbrace{\tfrac{1}{3}(\alpha I_b + \alpha^2 I_c)}^{I_a^+} - \overbrace{\tfrac{1}{3}(\alpha^2 I_b + \alpha I_c)}^{I_a^-}$$

$$\tfrac{1}{3}([\alpha - \alpha^2] I_b + [\alpha^2 - \alpha] I_c])$$

$$\tfrac{1}{3}((\alpha-\alpha^2) I_b + (-(\alpha-\alpha^2) I_c)) \rightarrow I_b = -I_c$$

$I_a^+ = -I_a^- \quad \therefore \quad I_a^+ + I_a^- = 0$

$I_a^+ = -I_a^- = \frac{1}{3}(\alpha - \alpha^2) I_b$

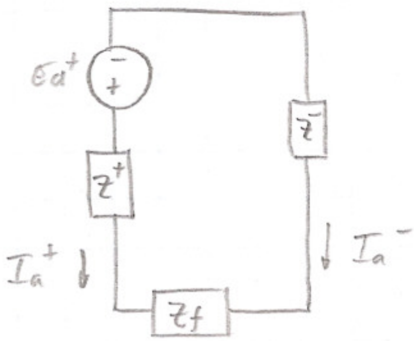
Voltage condition gives:

$$(\alpha^2 - \alpha)(V_a^+ - V_a^-) = Z_f(\alpha^2 - \alpha) I_a^+$$

which reduces to

$$V_a^+ - V_a^- = Z_f I_a^+$$

The equivalent cct. is:



⇒ Power in terms of symmetrical components...

Complex power (3ϕ) is given by

$$\bar{S}_{3\phi} = P_{3\phi} + j\,Q_{3\phi}$$
$$= \bar{V}_a \bar{I}_a^* + \bar{V}_b \bar{I}_b^* + \bar{V}_c \bar{I}_c^*$$
$$= \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix}$$

But we have

$$\underline{V_s} = \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}, \quad \underline{I_s} = \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

and $\underline{V} = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix}^T$ $\quad I = \begin{bmatrix} I_a & I_b & I_c \end{bmatrix}^T$

From $\underline{V} = \underline{A}\,\underline{V_s}$ and $\underline{I} = \underline{A}\,\underline{I_s}$, then:

$$\bar{S}_{3\phi} = (\underline{V})^T \cdot \underline{I}^*$$
$$= [\underline{A}\underline{V_s}]^T [\underline{A}\underline{I}]^*$$

$$[A V_s]^T = [V_s]^T [A]^T$$

$$(\underline{A} I_s)^* = A^* I_s^*$$

$$\bar{S}_{3\phi} = V_s^T A^T A^* I_s^*$$

$$\underline{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$\underline{A}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

$$[A][A^*] = \frac{1}{3}.$$